Pearson Edexcel

Mark Scheme (Results)

January 2021

Pearson Edexcel IAL Mathematics
Pure Mathematics P4
Paper WMA14/01

Question Number	Scheme	Marks
1(a)	$\left(\frac{1}{4}-5x\right)^{\frac{1}{2}}=\frac{1}{2}(...)$	B1
	$=(1-20x)^{\frac{1}{2}}=1+\left(\frac{1}{2}\right)\times(-20x)+\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{2!}\times(-20x)^2+\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{3!}\times(-20x)^3...$	M1A1
	$=\frac{1}{2}-5x-25x^2-250x^3+...$	A1 A1
	Special case: If the final answer is left as $\frac{1}{2}\left(1-10x-50x^2-500x^3+...\right)$ Award SC B1M1A1A1A0	
		(5)
	Alternative by direct expansion	
	$\left(\frac{1}{4}-5x\right)^{\frac{1}{2}}=\left(\frac{1}{4}\right)^{\frac{1}{2}}+\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)^{-\frac{1}{2}}(-5x)^1+\frac{\frac{1}{2}\times-\frac{1}{2}}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}(-5x)^2+\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\left(\frac{1}{4}\right)^{-\frac{5}{2}}(-5x)^3$	B1M1A1
	$=\frac{1}{2}-5x-25x^2-250x^3+...$	A1A1
(b)	$\left(\frac{1}{4}-\frac{5}{100}\right)^{\frac{1}{2}}=\left(\frac{1}{5}\right)^{\frac{1}{2}}=\frac{1}{2}-5\times\frac{1}{100}-25\left(\frac{1}{100}\right)^2-250\left(\frac{1}{100}\right)^3+...$ $\frac{\sqrt{5}}{5}\approx\frac{1789}{4000}$ or $\frac{1}{\sqrt{5}}\approx\frac{1789}{4000}$ $\Rightarrow\sqrt{5}\approx5\times\frac{1789}{4000}$ or $\sqrt{5}\approx1\div\frac{1789}{4000}$	M1
	$\sqrt{5}\approx\frac{1789}{800}$ or $\frac{4000}{1789}$	A1
		(2)
		(7 marks)

(a)

B1: For taking out a factor of $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ or $\frac{1}{2}$ or 0.5 etc.

M1: Expands $(1+kx)^{\frac{1}{2}}$, $k\neq\pm1$ with the correct structure for the third or fourth term

e.g. $\pm\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\times(kx)^2$ or $\pm\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\times(kx)^3$ with or without the bracket around the kx

A1: For either term three or term four being correct in any form.

E.g. $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\times(20x)^2$ or $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\times(-20x)^2$ or $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\times(-20x)^3$ or $-\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\times(20x)^3$

The brackets must be present unless they are implied by subsequent work. This mark is independent of the B mark.

A1: Two terms correct and simplified of $\frac{1}{2}-5x-25x^2-250x^3$. Allow if any of the '–' signs are written as "+–".

A1: All four terms correct and simplified of $\frac{1}{2}-5x-25x^2-250x^3$. Allow the terms to be listed.

Ignore any extra terms and apply isw if necessary. If any of the '–' signs are written as "+–" score A0.

Alternative:

B1: For a first term of $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ or $\frac{1}{2}$ or 0.5 etc.

M1: For the correct structure for the third or fourth term. E.g. $\frac{\frac{1}{2}\times-\frac{1}{2}}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}(kx)^2$ or $\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\left(\frac{1}{4}\right)^{-\frac{5}{2}}(kx)^3$

where $k\neq\pm1$

A1: For either term three or term four being correct in any form.

$$\text{e.g. } \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\times\left(\frac{1}{4}\right)^{-\frac{3}{2}}(-5x)^2 \text{ or } \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\times\left(\frac{1}{4}\right)^{-\frac{5}{2}}(-5x)^3$$

The brackets must be present unless they are implied by subsequent work.

A1: Two terms correct and simplified of $\frac{1}{2}-5x-25x^2-250x^3$. Allow if any of the '–' signs are written as "+–".

A1: All four terms correct and simplified of $\frac{1}{2}-5x-25x^2-250x^3$. Allow the terms to be listed.

Ignore any extra terms and apply isw if necessary. If any of the '–' signs are written as "+–" score A0.

(b)

M1: Attempts to substitute $x=\frac{1}{100}$ into their part (a) and either multiplies by 5 or finds reciprocal.

A1: $\left(\sqrt{5}=\right)\frac{1789}{800}$ or $\frac{4000}{1789}$

Question Number	Scheme	Marks
2(a)	$\overrightarrow{BA}.\overrightarrow{BC} = -6\times2+2\times5-3\times8 = (-26)$	M1
	Uses $\overrightarrow{BA}.\overrightarrow{BC} = \lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert\cos\theta \Rightarrow -26 = \sqrt{49}\times\sqrt{93}\cos\theta \Rightarrow \theta = ...$	**d**M1
	$\theta = 112.65°$	A1
		(3)
(b)	Attempts to use $\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert\sin\theta$ with their θ	M1
	Area = awrt 62.3	A1
		(2)
		(5 marks)

(a)

M1: Attempts the scalar product of $\pm\overrightarrow{AB}\,.\,\pm\overrightarrow{BC}$ condone slips as long as the intention is clear

Or attempts the vector product $\pm\overrightarrow{AB}\times\pm\overrightarrow{BC}$ **(see alternative 1)**

Or attempts vector AC **(see alternative 2)**

dM1: Attempts to use $\pm\overrightarrow{AB}.\overrightarrow{BC} = \lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{BC}\rvert\cos\theta$ AND proceeds to a value for θ

Expect to see at least one correct attempted calculation for a modulus.

For example $\sqrt{2^2+5^2+8^2}\left(=\sqrt{93}\right)$ or $\sqrt{6^2+2^2+3^2}\left(=7\right)$

Note that we condone poor notation such as: $\cos\theta = \dfrac{26}{7\sqrt{93}} = 67.35°$ **Depends on the first mark.**

<u>**Must be an attempt to find the correct angle.**</u>

A1: $\theta =$ awrt $112.65°$ Versions finishing with $\theta =$ awrt $67.35°$ will normally score M1 dM1 A0

Angles given in radians also score A0 (NB $\theta = 1.9661...$ or acute $1.1754...$)

Allow e.g. $\theta = 67.35° \Rightarrow \theta = 180-67.35° = 112.65$ and allow $\cos\theta = \dfrac{26}{7\sqrt{93}} \Rightarrow \theta = 112.65$

<u>**1. Alternative using the vector product:**</u>

M1: Attempts the vector product $\pm\overrightarrow{AB}\times\pm\overrightarrow{BC} = \pm\begin{pmatrix}6\\-2\\3\end{pmatrix}\times\pm\begin{pmatrix}2\\5\\8\end{pmatrix} = \pm\begin{pmatrix}-31\\-42\\34\end{pmatrix}$ condone slips as long as the intention is clear

dM1: Attempts to use $\pm\overrightarrow{AB}\times\overrightarrow{BC} = \lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{BC}\rvert\sin\theta$ AND proceeds to a value for θ

Expect to see at least one correct attempted calculation for a modulus on rhs and attempt at the modulus of the vector product

For example $\sqrt{2^2+5^2+8^2}$ or $\sqrt{6^2+2^2+3^2}$ and $\sqrt{31^2+42^2+34^2}\left(=\sqrt{3881}\right)$

Note that we condone poor notation such as: $\sin\theta = \frac{\sqrt{3881}}{7\sqrt{93}} = 67.35°$ **Depends on the first mark.**

<u>Must be an attempt to find the correct angle.</u>

A1: $\theta = \text{awrt } 112.65°$ Versions finishing with $\theta = \text{awrt } 67.35°$ will normally score M1 dM1 A0

<u>2. Alternative using cosine rule:</u>

M1: Attempts $\pm\overrightarrow{AC} = \pm\left(\overrightarrow{AB} + \overrightarrow{BC}\right) = \pm(8\mathbf{i} + 3\mathbf{j} + 11\mathbf{k})$ condone slips and poor notation as long as the intention is clear e.g. allow $\begin{pmatrix} 8\mathbf{i} \\ 3\mathbf{j} \\ 11\mathbf{k} \end{pmatrix}$

dM1: Attempts to use $AC^2 = AB^2 + BC^2 - 2AB.BC\cos\theta$ AND proceeds to a value for θ

<u>Must be an attempt to find the correct angle.</u>

A1: $\theta = \text{awrt } 112.65°$

(b)

M1: Attempts to use $\left|\overrightarrow{AB}\right|\left|\overrightarrow{BC}\right|\sin\theta$ with their θ. You may see $\frac{1}{2}\left|\overrightarrow{AB}\right|\left|\overrightarrow{BC}\right|\sin\theta$ found first before it is doubled.

or attempts the magnitude of their vector product e.g. $\sqrt{3881}$

A1: Area = awrt 62.3. If this is achieved from an angle of $\theta = \text{awrt } 67.35°$ full marks can be scored

Note that there are other more convoluted methods for finding the area – score M1 for a complete and correct method using their values and send to review if necessary.

Question Number	Scheme	Marks
3	States the largest odd number and an odd number that is greater E.g. odd number n and $n + 2$	M1
	Fully correct proof including • the assumption: there exists a greatest odd number "n" • a correct statement that their second odd number is greater than their assumed greatest odd number • a minimal conclusion " this is a contradiction, hence proven" You can ignore any spurious information e.g. $n > 0$, $n + 2 > 0$ etc.	A1*
		(2)
		(2 marks)

M1: For starting the proof by **stating** an odd number and a larger odd number.
Examples of an allowable start are

- **odd number** "n" with "$n + 2$"
- **odd number** "n" with "n^2"
- "$2k + 1$" with "$2k + 3$"
- "$2k + 1$" with "$(2k + 1)^3$"
- "$2k + 1$" with "$2k + 1 + 2k$"

Note that stating $n = 2k$, even when accompanied by the statement that "n" is odd is M0

A1*: A fully correct proof using contradiction
This must consist of

1) An assumption E.g. "(Assume that) there exists a greatest odd number n"
"Let "$2k + 1$" be the greatest odd number"

2) A minimal statement showing their second number is greater than the first,
E.g. If "n" is odd and "$n + 2$" is greater than n

If "n" is odd and $n^2 > n$

$2k+3 > 2k+1$

$2k+2k+1 > 2k+1$

Any algebra (e.g. expansions) must be correct. So $(2k+1)^2 = 4k^2 + 2k + 1$ would be A0

3) A minimal conclusion which could be
"hence there is no greatest odd number", "hence proven", or simply ✓

Question Number	Scheme	Marks
4(a)	$k=2$ or $x>2$	B1
	$t=\dfrac{1}{x-2}\Rightarrow y=\dfrac{1-\frac{2}{x-2}}{3+\frac{1}{x-2}}$	M1 A1
	$\dfrac{1-\frac{2}{x-2}}{3+\frac{1}{x-2}}=\dfrac{x-2-2}{\ldots}$ or $\dfrac{\ldots}{3(x-2)+1}$	A1 (M1 on EPEN)
	$y=\dfrac{x-4}{3x-5}$	A1
		(5)
(b)	$-2<\mathrm{g}<\dfrac{1}{3}$	M1 A1
		(2)
		(7 marks)

(a)

B1: States that $k = 2$ or else states that the domain is $x > 2$. **Must be seen in part (a).**

M1: Attempts to find t in terms of x and substitutes into y.

Condone poor attempts but you should expect to see $t = \mathrm{f}(x)$ found from $x=\dfrac{1}{t}+2$ substituted into $y=\dfrac{1-2t}{3+t}$ condoning slips.

A1: A correct unsimplified equation involving just x and y

A1(M1 on EPEN): Correct numerator or denominator with fraction removed (allow unsimplified)

A1: $y=\dfrac{x-4}{3x-5}$ or $\mathrm{g}(x)=\dfrac{x-4}{3x-5}$ (must be $y = \ldots$ or $\mathrm{g}(x) = \ldots$ but allow this mark as long as the $y = \ldots$ or $\mathrm{g}(x) = \ldots$ is present at some point)

Alternative 1 for part (a)

M1: Assume $\mathrm{g}(x)=\dfrac{ax+b}{cx+d}$ and substitute in $x=\dfrac{1}{t}+2$

A1: $\mathrm{g}(x)=\dfrac{a+(b+2a)t}{c+(d+2c)t}$

A1(M1 on EPEN): Correct numerator or denominator

A1: $y=\dfrac{x-4}{3x-5}$ or $\mathrm{g}(x)=\dfrac{x-4}{3x-5}$ (must be $y = \ldots$ or $\mathrm{g}(x) = \ldots$ but allow this mark as long as the $y = \ldots$ or $\mathrm{g}(x) = \ldots$ is present at some point)

Alternative 2 for part (a)

M1: Attempts to find t in terms of y and substitutes into x.

Condone poor attempts but you should expect to see $t = \mathrm{f}(y)$ found from $y = \dfrac{1-2t}{3+t}$ substituted into $x = \dfrac{1}{t} + 2$ condoning slips. (NB $t = \dfrac{1-3y}{y+2} \Rightarrow x = \dfrac{y+2}{1-3y} + 2$)

A1: A correct unsimplified equation involving just x and y

A1(M1 on EPEN): Correct numerator or denominator

A1: $y = \dfrac{x-4}{3x-5}$ or $\mathrm{g}(x) = \dfrac{x-4}{3x-5}$ (must be $y = \ldots$ or $\mathrm{g}(x) = \ldots$ but allow this mark as long as the $y = \ldots$ or $\mathrm{g}(x) = \ldots$ is present at some point)

(b)

M1: For obtaining one of the 2 boundaries (just look for values) e.g. -2 or $\dfrac{1}{3}$ or for attempting g(2) for their g **or** for attempting $\dfrac{\text{their } a}{\text{their } c}$. Note that for this mark they must be attempting values of y (or $\mathrm{g}(x)$).

A1: Correct range: Allow $-2 < \mathrm{g} < \dfrac{1}{3}$, $-2 < \mathrm{g}(x) < \dfrac{1}{3}$, $-2 < y < \dfrac{1}{3}$, $\left(-2, \dfrac{1}{3}\right)$, $\mathrm{g} > -2$ **and** $\mathrm{g} < \dfrac{1}{3}$

Question Number	Scheme	Marks
5	$u=3+\sqrt{2x-1}\Rightarrow x=\dfrac{(u-3)^2+1}{2}\Rightarrow\dfrac{dx}{du}=u-3$ **or** $u=3+\sqrt{2x-1}\Rightarrow\dfrac{du}{dx}=\dfrac{1}{2}(2x-1)^{-\frac{1}{2}}\times 2=\dfrac{1}{\sqrt{2x-1}}=\dfrac{1}{u-3}$	M1 A1
	$\displaystyle\int\frac{4}{3+\sqrt{2x-1}}\,dx=\int\frac{4}{u}\times(u-3)\,du$	M1
	$\displaystyle\int\frac{4}{u}\times(u-3)\,du=\int\left(4-\frac{12}{u}\right)du$	**d**M1
	$\displaystyle\int\left(4-\frac{12}{u}\right)du=4u-12\ln u$ or $k(4u-12\ln u)$	**dd**M1 A1ft
	$\displaystyle\int_1^{13}\frac{4}{3+\sqrt{2x-1}}\,dx=[4u-12\ln u]_4^8=(4\times8-12\ln8)-(4\times4-12\ln4)$ **or** $\displaystyle\int_1^{13}\frac{4}{3+\sqrt{2x-1}}\,dx=\left[4\left(3+\sqrt{2x-1}\right)-12\ln\left(3+\sqrt{2x-1}\right)\right]_1^{13}=(4\times8-12\ln8)-(4\times4-12\ln4)$	M1
	$=16-12\ln2$	A1
		(8 marks)

M1: Differentiates to get $\dfrac{du}{dx}$ in terms of x and then obtains $\dfrac{dx}{du}$ in terms of u

Need to see $\dfrac{du}{dx}=k(2x-1)^{-\frac{1}{2}}\rightarrow\dfrac{du}{dx}=\dfrac{1}{au+b}$ or $\dfrac{dx}{du}=au+b$

or

Attempts to change the subject of $u=3+\sqrt{2x-1}$ and differentiates to get $\dfrac{dx}{du}$ in terms of u

Need to see $x=\dfrac{(u\pm3)^2\pm1}{2}\rightarrow\dfrac{dx}{du}=au+b$

A1: $\dfrac{dx}{du}=u-3$ oe e.g. $\dfrac{du}{dx}=\dfrac{1}{u-3}$, $du=\dfrac{dx}{u-3}$, $dx=(u-3)du$

M1: Attempts to write the integral completely in terms of u.

Need to see $\displaystyle\int\frac{\dots}{u}\times\text{their}\frac{dx}{du}\,du$ with or without the "du" but **not** e.g. $\displaystyle\int\frac{\dots}{u}\times\frac{1}{\frac{dx}{du}}\,du$

dM1: Divides to reach an integral of the form $\displaystyle\int\left(A+B\times\frac{1}{u}\right)du$. **Depends on both previous M's**

dM1: Integrates to a form $Au+B\ln u$. **Depends on the previous M.**

An alternative for the previous 2 marks is to use integration by parts:

E.g. $\int \frac{4}{u} \times (u-3)\ \mathrm{d}u = 4(u-3)\ln u - \int 4\ln u\ \mathrm{d}u = 4u\ln u - 12\ln u - 4u\ln u + 4u = 4u - 12\ln u$

Score dM1 for $\int \frac{k}{u} \times (Au+B)\ \mathrm{d}u = k(Au+B)\ln u - \int k\ln u\ \mathrm{d}u$ and dM1 for integrating to a form $Au + B\ln u$.

A1ft: $4u - 12\ln u$ or $k(4u - 12\ln u)$ following through on $\frac{\mathrm{d}x}{\mathrm{d}u} = k(u-3)$ only.

M1: Substitutes 8 and 4 into their $4u - 12\ln u$ and subtracts **or** substitutes 13 and 1 into their $4u - 12\ln u$ with $u = 3 + \sqrt{2x-1}$ and subtracts. This mark depends on there having been an attempt to integrate, however poor.

A1: $16 - 12\ln 2$

Question Number	Scheme	Marks
6(a)	$4y^2+3x=6y\mathrm{e}^{-2x}$	
	$4y^2+3x \to 8y\dfrac{\mathrm{d}y}{\mathrm{d}x}+3$	B1
	$6y\mathrm{e}^{-2x} \to -12y\mathrm{e}^{-2x}+6\mathrm{e}^{-2x}\dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1 A1
	$8y\dfrac{\mathrm{d}y}{\mathrm{d}x}+3=-12y\mathrm{e}^{-2x}+6\mathrm{e}^{-2x}\dfrac{\mathrm{d}y}{\mathrm{d}x} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{12y\mathrm{e}^{-2x}+3}{6\mathrm{e}^{-2x}-8y}$ oe	M1 A1
		(5)
(b)	Sets $x=0$ in $4y^2+3x=6y\mathrm{e}^{-2x} \Rightarrow y=\dfrac{3}{2}$ oe	B1
	Substitutes $\left(0,\dfrac{3}{2}\right)$ in their $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{12y\mathrm{e}^{-2x}+3}{6\mathrm{e}^{-2x}-8y}=\left(\dfrac{7}{-2}\right)$	M1
	$m_N=-1\div"\dfrac{7}{-2}" \Rightarrow y="\dfrac{2}{7}"x+"\dfrac{3}{2}"$	**d**M1
	$y=\dfrac{2}{7}x+\dfrac{3}{2}$ oe e.g. $y=\dfrac{6}{21}x+\dfrac{3}{2}$	A1
		(4)
		(9 marks)

(a)

B1: Differentiates $4y^2+3x$ to obtain $8y\dfrac{\mathrm{d}y}{\mathrm{d}x}+3$. Allow unsimplified forms such as $4\times 2y\dfrac{\mathrm{d}y}{\mathrm{d}x}+3$

M1: Uses the product rule on $6y\mathrm{e}^{-2x}$ to obtain an expression of the form $Ay\mathrm{e}^{-2x}+B\mathrm{e}^{-2x}\dfrac{\mathrm{d}y}{\mathrm{d}x}$

A1: Differentiates $6y\mathrm{e}^{-2x}$ to obtain $-12y\mathrm{e}^{-2x}+6\mathrm{e}^{-2x}\dfrac{\mathrm{d}y}{\mathrm{d}x}$

M1: Collects two terms in $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ (one from attempting to differentiate $4y^2$ and one from attempting to differentiate $6y\mathrm{e}^{-2x}$) and proceeds to make $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ the subject.

A1: $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{12y\mathrm{e}^{-2x}+3}{6\mathrm{e}^{-2x}-8y}$ or equivalent e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2\mathrm{e}^{-2x}\times 6y+3}{6\mathrm{e}^{-2x}-8y}$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{12y+3\mathrm{e}^{2x}}{6-8y\mathrm{e}^{2x}}$

You can ignore any spurious $"\dfrac{\mathrm{d}y}{\mathrm{d}x}="$ at the start and allow y' for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(b)

B1: Uses $x = 0$ to obtain $y=\dfrac{3}{2}$ oe e.g. $\dfrac{6}{4}$ (ignore any reference to $y = 0$)

M1: Substitutes $x = 0$ and their y at $x = 0$ which has come from substituting $x = 0$ into the original equation into their $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{12y\mathrm{e}^{-2x}+3}{6\mathrm{e}^{-2x}-8y}$ to find a numerical value. Working is normally shown here but you may need to check for evidence. Use of $x = 0$ and $y = 0$ is M0.

dM1: Uses the negative reciprocal of $"\frac{7}{-2}"$ for the gradient of the normal and uses this and their value of y at $x = 0$ to form the equation of the normal. **Depends on the previous M.**

A1: $y = \frac{2}{7}x + \frac{3}{2}$ oe e.g. $y = \frac{6}{21}x + \frac{6}{4}$

Note that the use of (0, 0) for P will generally lose the final 3 marks in (b)

Question Number	Scheme	Marks
7(a) Way 1	$\int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \int \frac{1}{2}e^{2x}\cos x\,dx$	M1
	$= ... - \frac{1}{4}e^{2x}\cos x - \int \frac{1}{4}e^{2x}\sin x\,dx$	**d**M1
	$\int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x - \int \frac{1}{4}e^{2x}\sin x\,dx$	A1
	$\frac{5}{4}\int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x \Rightarrow \int e^{2x}\sin x\,dx = ...$	**dd**M1
	$= \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x + c$	A1
		(5)
7(a) Way 2	$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + \int 2e^{2x}\cos x\,dx$	M1
	$= ... + 2e^{2x}\sin x - \int 4e^{2x}\sin x\,dx$	**d**M1
	$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x - \int 4e^{2x}\sin x\,dx$	A1
	$5\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x \Rightarrow \int e^{2x}\sin x\,dx = ...$	**dd**M1
	$= \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x + c$	A1
		(5)
(b)	$\left(\frac{2}{5}e^{2\pi}\sin\pi - \frac{1}{5}e^{2\pi}\cos\pi\right) - \left(\frac{2}{5}e^{0}\sin 0 - \frac{1}{5}e^{0}\cos 0\right) = ...$	M1
	$= \frac{1}{5}e^{2\pi} + \frac{1}{5} = \frac{e^{2\pi}+1}{5}$ *	A1*
		(2)
		(7 marks)

Note that you can condone the omission of the d*x*'s throughout.

(a) Way 1

M1: Attempts integration by parts with $u = \sin x$ and $v' = e^{2x}$ to obtain

$$\int e^{2x}\sin x\,dx = Ae^{2x}\sin x \pm B\int e^{2x}\cos x\,dx \quad A > 0$$

dM1: Attempts integration by parts again with $u = \cos x$ and $v' = e^{2x}$ on $B\int e^{2x}\cos x\,dx$ to obtain

$$B\int e^{2x}\cos x\,dx = \pm Ce^{2x}\cos x \pm D\int e^{2x}\sin x\,dx$$

<u>Depends on the previous mark.</u>

A1: For $\int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x - \int \frac{1}{4}e^{2x}\sin x\,dx$

Allow unsimplified e.g. $\int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \left\{\frac{1}{4}e^{2x}\cos x + \int \frac{1}{4}e^{2x}\sin x\,dx\right\}$

ddM1: **<u>Dependent upon having scored both M's.</u>**

It is for collecting $\int e^{2x}\sin x\,dx$ terms together and making it the subject of the formula

A1: $\int e^{2x}\sin x\,dx = \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x + c$ (allow with or without "+ c")

(a) **Way 2**

M1: Attempts integration by parts with $u = e^{2x}$ and $v' = \sin x$ to obtain

$$\int e^{2x}\sin x\,dx = \pm Ae^{2x}\cos x \pm B\int e^{2x}\cos x\,dx$$

dM1: Attempts integration by parts again with $u = e^{2x}$ and $v' = \cos x$ on $B\int e^{2x}\cos x\,dx$ to obtain

$$B\int e^{2x}\cos x\,dx = \pm Ce^{2x}\sin x \pm D\int e^{2x}\sin x\,dx$$

<u>Depends on the previous mark.</u>

A1: For $\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x - \int 4e^{2x}\sin x\,dx$

Allow unsimplified e.g. $\int e^{2x}\sin x\,dx = -e^{2x}\cos x - \left\{-2e^{2x}\sin x - \int -4e^{2x}\sin x\,dx\right\}$

ddM1: **<u>Dependent upon having scored both M's.</u>**

It is for collecting $\int e^{2x}\sin x\,dx$ terms together and making it the subject of the formula

A1: $\int e^{2x}\sin x\,dx = \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x + c$ (allow with or without "+ c")

(a) **Way 3**

M1: Attempts integration by parts with $u = \sin x$ and $v' = e^{2x}$ to obtain

$\int e^{2x}\sin x\,dx = Ae^{2x}\sin x \pm B\int e^{2x}\cos x\,dx \quad A > 0$

or attempts integration by parts with $u = e^{2x}$ and $v' = \sin x$ to obtain

$\int e^{2x}\sin x\,dx = \pm Ae^{2x}\cos x \pm B\int e^{2x}\cos x\,dx$

dM1: Attempts integration by parts with $u = \sin x$ and $v' = e^{2x}$ to obtain

$\int e^{2x}\sin x\,dx = \pm Ae^{2x}\sin x \pm B\int e^{2x}\cos x\,dx$

and attempts integration by parts with $u = e^{2x}$ and $v' = \sin x$ to obtain

$\int e^{2x}\sin x\,dx = \pm Ae^{2x}\cos x \pm B\int e^{2x}\cos x\,dx$

A1: $I_1 = \int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \int \frac{1}{2}e^{2x}\cos x\,dx$ **AND** $I_2 = \int e^{2x}\sin x\,dx = -e^{2x}\cos x + \int 2e^{2x}\cos x\,dx$

ddM1: E.g. $4I_1 + I_2 = 2e^{2x}\sin x - e^{2x}\cos x = 5I \Rightarrow I = ...$ Correct attempt to eliminate $\int e^{2x}\cos x\,dx$ term.

A1: $\int e^{2x}\sin x\,dx = \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x + c$ (allow with or without "+ c")

(b)

M1: For applying the limits 0 and π to an expression containing at least one term of the form $Ae^{2x}\sin x$ and at least one term of the form $Be^{2x}\cos x$. **There must be some evidence that <u>both</u> limits have been used.**

A1*: $\frac{e^{2\pi}+1}{5}$ found correctly **<u>from the correct answer in part (a)</u>** via at least one intermediate line which could be $\frac{e^{2\pi}}{5}+\frac{1}{5}$

Note a correct answer in (a) and evidence of use of the limits 0 and pi followed by $\frac{e^{2\pi}+1}{5}$ with no intermediate line scores M1A0

Question Number	Scheme	Marks
8	$\begin{pmatrix}-1\\5\\4\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\5\end{pmatrix}=\begin{pmatrix}2\\-2\\-5\end{pmatrix}+\mu\begin{pmatrix}4\\-3\\b\end{pmatrix}\Rightarrow \begin{array}{l}-1+2\lambda=2+4\mu \ (1)\\ 5-\lambda=-2-3\mu \ (2)\\ 4+5\lambda=-5+\mu b \ (3)\end{array}$	
	Uses equations (1) and (2) to find either λ **or** μ e.g. $(1)+2(2)\Rightarrow \mu=...$ or $3(1)+4(2)\Rightarrow \lambda=...$	M1
	Uses equations (1) and (2) to find both λ and μ	**d**M1
	$\mu=-\frac{11}{2}$ and $\lambda=-\frac{19}{2}$	A1
	$4+5\lambda=-5+\mu b\Rightarrow 4+5\times-\frac{19}{2}=-5-\frac{11}{2}b$ or $4+5\lambda=-5+7\mu\Rightarrow 4+5\times-\frac{19}{2}=-5-\frac{11}{2}\times 7$	**dd**M1
	$\Rightarrow 11b=77\Rightarrow b=7$ or obtains $-\frac{87}{2}=-\frac{87}{2}$	A1
	States that when $b=7$, lines intersect or when $b\neq 7$, lines do not intersect Lines are not parallel so when $b\neq 7$ lines are skew. *	A1 **Cso**
		(6)
	Alternative assuming $b = 7$:	
	$\begin{pmatrix}-1\\5\\4\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\5\end{pmatrix}=\begin{pmatrix}2\\-2\\-5\end{pmatrix}+\mu\begin{pmatrix}4\\-3\\7\end{pmatrix}\Rightarrow \begin{array}{l}-1+2\lambda=2+4\mu \ (1)\\ 5-\lambda=-2-3\mu \ (2)\\ 4+5\lambda=-5+7b \ (3)\end{array}$	
	Uses any 2 equations to find either λ **or** μ	M1
	Uses any 2 equations to find both λ and μ	**d**M1
	$\mu=-\frac{11}{2}$ and $\lambda=-\frac{19}{2}$	A1
	Checks in the 3rd equation e.g. equation 3: $4+5\left(-\frac{19}{2}\right)=-5+7\left(-\frac{11}{2}\right)=...$ equation 1: $-1+2\left(-\frac{19}{2}\right)=2+4\left(-\frac{11}{2}\right)=...$ equation 2: $5-\left(-\frac{19}{2}\right)=-2-3\left(-\frac{11}{2}\right)=...$	**dd**M1
	Equation 3: $-\frac{87}{2}$ Equation 1: -20 Equation 2: $\frac{29}{2}$	A1
	States that when $b=7$, lines intersect or when $b\neq 7$, lines do not intersect Lines are not parallel so when $b\neq 7$ lines are skew. *	A1 **Cso**
		(6 marks)

M1: For attempting to solve equations (1) and (2) to find **either** λ **or** μ

dM1: For attempting to solve equations (1) and (2) to find **both** λ **and** μ **Depends on the first M.**

A1: $\mu = -\frac{11}{2}$ and $\lambda = -\frac{19}{2}$

ddM1: Attempts to solve $4 + 5\lambda = -5 + \mu b$ for their values of λ and μ. Or uses $b = 7$ with their λ and μ in an attempt to show equality. **Depends on both previous M's.**

A1: Achieves (without errors) that they will intersect when $b = 7$

Note that the previous 3 marks may be scored without explicitly seeing the values of both parameters e.g.

$\mu = -\frac{11}{2}$, (2) → $\lambda = 3\mu + 7$ → $4 + 5(3\mu + 7) = -5 + \mu b$ → $b = 7$

A1*:**Cso** States that when $b = 7,$ lines intersect and since lines are not parallel it shows that when $b \neq 7$ lines are skew.

Alternative:

M1: Uses $b = 7$ and attempts to solve 2 equations to find **either** λ **or** μ

dM1: For attempting to solve 2 equations to find **both** λ **and** μ **Depends on the first M.**

A1: $\mu = -\frac{11}{2}$ and $\lambda = -\frac{19}{2}$

ddM1: Attempts to show that the 3rd equation is true for their values of λ and μ

Depends on both previous M's.

A1: Achieves (without errors) that the 3rd equation gives the same values for (or equivalent)

A1*: **Cso** States that when $b = 7,$ lines intersect and since lines are not parallel it shows that when $b \neq 7$ lines are skew.

To score the final mark there must be some statement that the lines intersect (or equivalent e.g. meet at a point, cross, etc.) when $b = 7$ or that they do not intersect if $b \neq 7$ **and** that the lines are not parallel which may appear anywhere (reason not needed but may be present) so lines are skew when $b \neq 7$.

Ignore any work attempting to show that the lines are perpendicular or not.

Question Number	Scheme	Marks
9(a)	$\tan\theta=\sqrt{3}\Rightarrow k=\frac{\pi}{3}(\text{or } 60°)$ (Allow $x=\sqrt{3}\Rightarrow\theta=\frac{\pi}{3}(\text{or } 60°)$)	B1
	$V=(\pi)\int y^2\mathrm{d}x=(\pi)\int(2\sin 2\theta)^2\sec^2\theta\,\mathrm{d}\theta$ oe	M1A1
	$4(\pi)\int\sin^2 2\theta\sec^2\theta\ \mathrm{d}\theta=4(\pi)\int 4\sin^2\theta\cancel{\cos^2\theta}\times\frac{1}{\cancel{\cos^2\theta}}\ \mathrm{d}\theta$	**d**M1
	$=16(\pi)\int\sin^2\theta\,\mathrm{d}\theta$ oe e.g. $16(\pi)\int(1-\cos^2\theta)\,\mathrm{d}\theta$	A1
	$\sin^2\theta=\frac{1-\cos 2\theta}{2}\Rightarrow 16(\pi)\int\sin^2\theta\ \mathrm{d}\theta=16(\pi)\int\frac{1-\cos 2\theta}{2}\ \mathrm{d}\theta$	**d**M1
	Volume $=\int_0^{\frac{\pi}{3}}8\pi(1-\cos 2\theta)\,\mathrm{d}\theta$	A1 **Cso**
		(7)
(b)	$\int(1-\cos 2\theta)\mathrm{d}\theta\to\theta-\frac{\sin 2\theta}{2}$	B1
	Volume $=\int_0^{\frac{\pi}{3}}8\pi(1-\cos 2\theta)\,\mathrm{d}\theta=[8\pi\theta-4\pi\sin 2\theta]_0^{\frac{\pi}{3}}=\frac{8}{3}\pi^2-2\sqrt{3}\pi$	M1 A1
		(3)
		(10 marks)

(a)

B1: States or uses $\tan\theta=\sqrt{3}\Rightarrow k=\frac{\pi}{3}$ (Allow 60° here). May be implied by their integral.

Allow if seen anywhere in the question either stated or used as their upper limit.

M1: Attempts volume = $(A\pi)\int y^2\mathrm{d}x=(A\pi)\int(2\sin 2\theta)^2\sec^2\theta\,\mathrm{d}\theta$ with or without π or "dθ".

Condone bracketing errors

A1: For a volume of $(A\pi)\int(2\sin 2\theta)^2\sec^2\theta\,\mathrm{d}\theta$ with or without π or "dθ". The brackets must be present but may be implied by subsequent work.

dM1: Uses $\sin 2\theta=2\sin\theta\cos\theta$ and proceeds to Volume = $B\int\sin^2\theta\ \mathrm{d}\theta$ with or without "dθ". (No requirement for limits yet). Note that if $(2\sin 2\theta)^2$ becomes $2\sin^2\theta\cos^2\theta$ with no evidence of a correct identity then score dM0

Depends on the first M.

A1: Volume $=(A\pi)\int 16\sin^2\theta\ \mathrm{d}\theta$ oe e.g. $(A\pi)\int 16(1-\cos^2\theta)\,\mathrm{d}\theta$ with or without π or "dθ". (No requirement for limits yet)

dM1: Attempts to use $\sin^2\theta=\frac{1-\cos 2\theta}{2}$ or $\cos^2\theta=\frac{1+\cos 2\theta}{2}$ and obtains Volume $=\int(P\pm Q\cos 2\theta)\ \mathrm{d}\theta$

Depends on the first M.

A1: **CSO** $\int_0^{\frac{\pi}{3}} 8\pi(1-\cos 2\theta)\,\mathrm{d}\theta$. Fully correct integral with both limits and the "dθ" but the 8 and/or the π can be either side of the integral sign.

Note this alternative solution for part (a):

$$V = (A\pi)\int y^2\mathrm{d}x = (A\pi)\int (2\sin 2\theta)^2 \sec^2\theta\,\mathrm{d}\theta = (A\pi)\int \frac{4\sin^2 2\theta}{\cos^2\theta}\mathrm{d}\theta$$ **M1 A1** as above

$$= (A\pi)\int \frac{4\sin^2 2\theta}{\frac{1}{2}(1+\cos 2\theta)}\mathrm{d}\theta$$

dM1: uses $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$ in the denominator. **A1:** Correct integral

$$= 8(A\pi)\int \frac{1-\cos^2 2\theta}{1+\cos 2\theta}\mathrm{d}\theta = 8(A\pi)\int \frac{(1+\cos 2\theta)(1-\cos 2\theta)}{1+\cos 2\theta}\mathrm{d}\theta$$

dM1: Uses $\sin^2 2\theta = 1-\cos^2 2\theta$ and the difference of 2 squares in the numerator and cancels

$$\text{Volume} = \int_0^{\frac{\pi}{3}} 8\pi(1-\cos 2\theta)\,\mathrm{d}\theta$$ **A1 CSO**

Note that a Cartesian approach in part (a) essentially follows the main scheme e.g.

$$V = (\pi)\int y^2\mathrm{d}x = (A\pi)\int (4x\cos^2\theta)^2 \sec^2\theta\,\mathrm{d}\theta = 4(A\pi)\int 4\sin^2\theta \cancel{\cos^2\theta} \times \frac{1}{\cancel{\cos^2\theta}}\,\mathrm{d}\theta \text{ etc.}$$

If in doubt whether such attempts deserve credit send to review.

(b)

B1: States or uses $\int (1-\cos 2\theta)\mathrm{d}\theta \to \theta - \dfrac{\sin 2\theta}{2}$

M1: Volume = $\int_0^{\frac{\pi}{3}} p(1-\cos 2\theta)\,\mathrm{d}\theta = \left[p\theta \pm kp\sin 2\theta\right]_0^{\frac{\pi}{3}}$ and uses the limit $\dfrac{\pi}{3}$ (not 60°).

(The limit of 0 may not be seen)

A1: $\dfrac{8}{3}\pi^2 - 2\sqrt{3}\pi$ oe e.g. $8\pi\left(\dfrac{\pi}{3}-\dfrac{\sqrt{3}}{4}\right)\dfrac{8}{3}$, $\pi^2 - 2\sqrt{3}\pi$, $\dfrac{2\pi}{3}\left(4\pi - 3\sqrt{3}\right)$, $\dfrac{8\pi^2 - 6\sqrt{3}\pi}{3}$

Question Number	Scheme	Marks
10(a)	$\dfrac{1}{(H-5)(H+3)}=\dfrac{A}{H-5}+\dfrac{B}{H+3}\Rightarrow A=\ldots$ or $B=\ldots$	M1
	$A=\dfrac{1}{8}$ or $B=-\dfrac{1}{8}$	A1
	$\dfrac{1}{(H-5)(H+3)}=\dfrac{1}{8(H-5)}-\dfrac{1}{8(H+3)}$ or $\dfrac{\frac{1}{8}}{(H-5)}-\dfrac{\frac{1}{8}}{(H+3)}$ or $\dfrac{\frac{1}{8}}{(H-5)}+\dfrac{-\frac{1}{8}}{(H+3)}$ or $\dfrac{1}{8H-40}-\dfrac{1}{8H+24}$	A1
		(3)
(b)	$\dfrac{dH}{dt}=-\dfrac{(H-5)(H+3)}{40}$ $\displaystyle\int\frac{40}{(H-5)(H+3)}dH=\int -1\,dt$ or e.g. $\displaystyle\int\frac{1}{(H-5)(H+3)}dH=\int-\frac{1}{40}\,dt$ $\displaystyle\int\frac{5}{(H-5)}-\frac{5}{(H+3)}dH=\int-1\,dt$ or e.g. $\displaystyle\frac{1}{8}\int\frac{1}{(H-5)}-\frac{1}{(H+3)}dH=\int-\frac{1}{40}\,dt$	M1
	$5\ln\lvert H-5\rvert-5\ln\lvert H+3\rvert=-t(+c)$ oe e.g. $\frac{1}{8}\ln\lvert H-5\rvert-\frac{1}{8}\ln\lvert H+3\rvert=-\frac{1}{40}t(+c)$ Or e.g. $5\ln(8H-40)-5\ln(8H+24)=-t(+c)$ etc.	M1 A1ft
	Substitutes $t=0, H=13\Rightarrow c=\ldots$ Note that this may happen at a later stage e.g. may attempt to remove logs and then substitute to find the constant	M1
	$5\ln\lvert H-5\rvert-5\ln\lvert H+3\rvert=-t+5\ln\left(\frac{1}{2}\right)$ oe e.g. $\frac{1}{8}\ln\lvert H-5\rvert-\frac{1}{8}\ln\lvert H+3\rvert=-\frac{1}{40}t+\frac{1}{8}\ln\left(\frac{1}{2}\right)$	A1
	$5\ln\left(2\left\lvert\dfrac{H-5}{H+3}\right\rvert\right)=-t\Rightarrow\dfrac{H-5}{H+3}=\dfrac{1}{2}e^{-0.2t}\Rightarrow H=\ldots$	**ddd**M1
	$H=\dfrac{10+3e^{-0.2t}}{2-e^{-0.2t}}$ *	A1*
		(7)
(c)	Sets $\dfrac{10+3e^{-0.2t}}{2-e^{-0.2t}}=8\Rightarrow e^{-0.2t}=\left(\dfrac{6}{11}\right)$	M1
	$\Rightarrow t=-5\ln\left(\dfrac{6}{11}\right)=$ awrt 3.03 days	**d**M1 A1
		(3)
(d)	$k=5$	B1
		(1)
		(14 marks)

(a)

M1: Attempts any correct method to find either constant. It is implied by one correct constant

A1: One correct constant

A1: Correct partial fractions: $\frac{1}{8(H-5)}-\frac{1}{8(H+3)}$. Note that this mark is not just for the correct constants, it is for the correctly stated fractions either in part (a) **or used in part (b).** Allow 0.125 for 1/8.

(b)

M1: Separates the variables and uses part (a) to reach: $\int \frac{P}{(H-5)}+\frac{Q}{(H+3)}\,\mathrm{d}H=\int \pm k\,\mathrm{d}t$ with or without the integral signs

M1: Attempts to integrate both sides to reach: $\alpha\ln|H-5|+\beta\ln|H+3|=kt$

or e.g. $\alpha\ln|8H-40|+\beta\ln|8H+24|=kt$ Condone $|\ | \leftrightarrow (\ )$ and condone the omission of brackets e.g. allow $\alpha\ln H-5+\beta\ln H+3=kt$ or e.g. $\alpha\ln 8H-40+\beta\ln 8H+24=kt$

A1ft: Correct integration of both sides following through on their PF in (a). Condone $|\ | \leftrightarrow (\ )$ and condone the omission of $+\,c$ but brackets must be present unless they are implied by subsequent work.

Also follow through on a MR of $\frac{\mathrm{d}H}{\mathrm{d}t}=\frac{(H-5)(H+3)}{40}$ for $\frac{\mathrm{d}H}{\mathrm{d}t}=-\frac{(H-5)(H+3)}{40}$

E.g. obtains $\frac{1}{8}\ln|H-5|-\frac{1}{8}\ln|H+3|=\frac{1}{40}t(+c)$

M1: Substitutes $t=0, H=13\Rightarrow c=...$ For this to be scored there must have been a $+\,c$ and depends on some attempt at integration of both sides however poor.

Alternatively attempts $\int_{13}^{H}\frac{1}{(H-5)}-\frac{1}{(H+3)}\,\mathrm{d}H=\int_0^t -\frac{1}{5}\,\mathrm{d}t\Rightarrow\left[\ln\frac{H-5}{H+3}\right]_{13}^{H}=\left[-\frac{1}{5}\right]_0^t\Rightarrow \ln\frac{H-5}{H+3}-\ln\frac{1}{2}=-\frac{1}{5}t$

A1: For a correct equation in H and t. Condone $|\ | \leftrightarrow (\ )$ but brackets must be present unless they are implied by subsequent work.

dddM1: A correct attempt to make H the subject of the formula. **<u>All previous M's in (b) must have been scored.</u>**

A1*: $H=\frac{10+3\mathrm{e}^{-0.2t}}{2-\mathrm{e}^{-0.2t}}$ cso with sufficient working shown and no errors.

Note that marks in (b) may need to be awarded retrospectively:

E.g. First 3 marks gained to reach $\ln|H-5|-\ln|H+3|=-\frac{1}{5}t+c$ and then:

$$\ln\frac{H-5}{H+3}=-\frac{1}{5}t+c\Rightarrow\frac{H-5}{H+3}=A\mathrm{e}^{-0.2t}\Rightarrow H=\frac{5+3A\mathrm{e}^{-0.2t}}{1-A\mathrm{e}^{-0.2t}}$$

$$H=13,\ t=0\Rightarrow 13=\frac{5+3A}{1-A}\Rightarrow A=\frac{1}{2}\Rightarrow H=\frac{5+\frac{3}{2}\mathrm{e}^{-0.2t}}{1-\frac{1}{2}\mathrm{e}^{-0.2t}}=\frac{10+3\mathrm{e}^{-0.2t}}{2-\mathrm{e}^{-0.2t}}\ *$$

The M3 can be awarded when they attempt to find "A", the dddM4 can be awarded for a correct attempt to make H the subject and then A2 and A3 can be awarded together at the end.

(c)

M1: Sets $\frac{10+3\mathrm{e}^{-0.2t}}{2-\mathrm{e}^{-0.2t}}=8$ or possibly an earlier version of H or possibly their t in terms of H and reaches $A\mathrm{e}^{\pm 0.2t}=p,\quad p>0$

dM1: Correct processing of an equation of the form $A\mathrm{e}^{\pm 0.2t}=p$ with correct log work leading to $t=...$

<u>Depends on the first M.</u>

A1: $t = -5\ln\left(\frac{6}{11}\right)$ or $t = 5\ln\left(\frac{11}{6}\right)$ or awrt 3.03 (days)

(d)

B1: $k = 5$ (Allow $H = 5$ or just "5")